

The answers start from the next page.

L 1-4

1	2
<p>1.</p> <p>(1) $\log_2 16 = 4$</p> <p>(2) $\log_5 125 = 3$</p> <p>(3) $\log_3 \frac{1}{9} = -2$</p> <p>(4) $\log_5 \sqrt{5} = \frac{1}{2}$</p> <p>2.</p> <p>(1) $3^2 = 9$</p> <p>(2) $5^0 = 1$</p> <p>(3) $2^{-4} = \frac{1}{16}$</p> <p>(4) $10^{-3} = 0.001$</p>	<p>1.</p> <p>(1) $x = 3$ (5) $x = 0$</p> <p>(2) $x = 4$ (6) $x = \frac{3}{2}$</p> <p>(3) $x = 3$ (7) $x = \frac{5}{3}$</p> <p>(4) $x = 1$ (8) $x = \frac{3}{2}$</p>
1	2
<p>3.</p> <p>(1) $x = 3$ (4) $x = -2$</p> <p>(2) $x = 6$ (5) $x = -5$</p> <p>(3) $x = 7$ (6) $x = \frac{1}{2}$</p> <p>(7) $x = -\frac{5}{2}$</p>	<p>2.</p> <p>(1) $x = 3$ (5) $x = 2$</p> <p>(2) $x = 4$ (6) $x = \frac{1}{9}$</p> <p>(3) $x = 32$ (7) $x = 0$</p> <p>(4) $x = 25$ (8) $x = -3$</p>

3	4
<p>(1) $x = 1$ (5) $x = 25$</p> <p>(2) $x = 0$ (6) $x = \frac{1}{4}$</p> <p>(3) $x = -1$ (7) $x = 5$</p> <p>(4) $x = -3$ (8) $x = 2$</p>	<p>(4) $x = \frac{1}{2}$</p> <p>(1) $x = 6$ (5) $x = \frac{1}{3}$</p> <p>(2) $x = -2$ (6) $x = \frac{3}{2}$</p> <p>(3) $x = -6$ (7) $x = 5$</p>
3	4
<p>(9) $x = 1$ (13) $x = 5$</p> <p>(10) $x = \frac{1}{2}$ (14) $x = \sqrt{5}$</p> <p>(11) $x = 0$ (15) $x = 3$</p> <p>(12) $x = -\frac{1}{2}$ (16) $x = \frac{1}{3}$</p>	<p>(8) $x = \frac{1}{3}$ (12) $x = \frac{5}{2}$</p> <p>(9) $x = -\frac{3}{2}$ (13) $x = 1000$</p> <p>(10) $x = 1$ (14) $x = \frac{1}{125}$</p> <p>(11) $x = \frac{1}{4}$ (15) $x = 25$</p>

L 5-8

5	6
<p>1.</p> <p>(1) \boxed{x} \boxed{y}</p> <p>(2) $\log_a x - \log_a y$</p> <p>(3) $-\log_a x$</p> <p>(4) $3\log_a x$</p>	<p>(1) $2a$</p> <p>(2) $\boxed{10}$ $1 - a$</p> <p>(3) $a + b$</p> <p>(4) $3a$</p> <p>(5) $2b$</p> <p>(6) $2a + b$</p> <p>(7) $a + 2b$</p>
5	6
<p>2.</p> <p>(1) $\log_a x + \log_a y + \log_a z$</p> <p>(2) $3\log_a x + 2\log_a y$</p> <p>(3) $\frac{2}{3}\log_a x$</p> <p>(4) $\frac{3}{2}\log_a x$</p> <p>(5) $\frac{2}{3}\log_a x + \frac{3}{2}\log_a y$</p> <p>(6) $\log_a x + \log_a y - \log_a z$</p> <p>(7) $2\log_a x - 4\log_a y - \log_a z$</p>	<p>(8) $2a + 2b$</p> <p>(9) $3a + 2b$</p> <p>(10) $a - b$</p> <p>(11) $b - a$</p> <p>(12) $1 - b$</p> <p>(13) $-a$</p> <p>(14) $-a - b$</p> <p>(15) $a + b - 1$</p>

7	8
<p>1.</p> <p>(1) $a+3b$</p> <p>(2) $b-2a$</p> <p>(3) $3a-2b$</p> <p>(4) $1-2a-b$</p> <p>(5) $a+\frac{b}{2}$</p> <p>(6) $\frac{1}{2}(1-a)$</p>	<p>1.</p> <p>(1) 3</p> <p>(2) $\frac{5}{3}$</p> <p>(3) $-\frac{1}{3}$</p>
7	8
<p>2.</p> <p>(1) 2</p> <p>(2) 4</p> <p>(3) 1</p> <p>(4) $\frac{3}{2}$</p>	<p>2.</p> <p>(1) 5</p> <p>(2) 2</p> <p>(3) 2</p>

L 9-12

9	10
<p>1.</p> <p>(1) 2</p> <p>(2) -5</p> <p>(3) 0</p>	<p>(1) 1</p> <p>(2) 3</p> <p>(3) 2</p>
9	10
<p>2.</p> <p>(1) 2</p> <p>(2) 1</p> <p>(3) 3</p>	<p>(4) 2</p> <p>(5) 4</p>

11	12
1. (1) $\frac{a+1}{a}$ (2) $\frac{1-a}{b}$ (3) 2	1. (1) $1-a$ (2) $2a+b$ (3) $\frac{b}{a}$ (4) $\frac{1-a}{a}$ (5) $\frac{a+2b}{a+b}$ (6) $\frac{3a-2b}{a+2b}$
11	12
2. (1) $1+a$ (2) $\frac{1}{a}$ (3) a (4) $\frac{1+a}{a}$ (5) $\frac{a}{1+a}$ (6) $a-1$	2. (1) $\frac{1}{ab}$ (2) $\frac{3+ab}{1+ab}$ (3) $\frac{2+ab}{1+a+ab}$

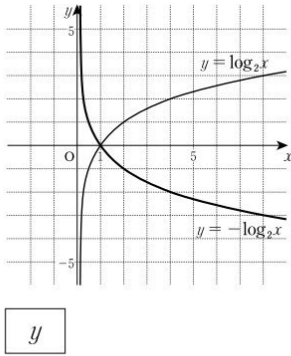
L 13–16

13	14
<p>1.</p> <p>(1) 1</p> <p>(2) $\frac{5}{4}$</p> <p>(3) 2</p> <p>(4) 1</p> <p>(5) 3</p>	<p>(1) 2</p> <p>(2) 2</p> <p>(3) 2</p>
13	14
<p>2.</p> <p>(1) 4</p> <p>(2) 4</p>	<p>(4) 5</p>

15

(1)

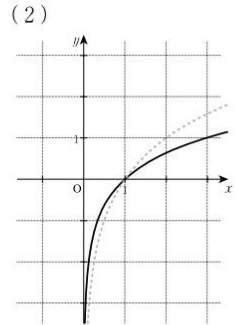
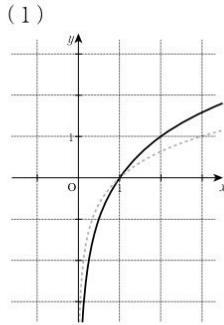
x	y
$\frac{1}{8}$	3
$\frac{1}{4}$	2
$\frac{1}{2}$	1
1	0
2	-1
4	-2
8	-3



y

16

1.



2.

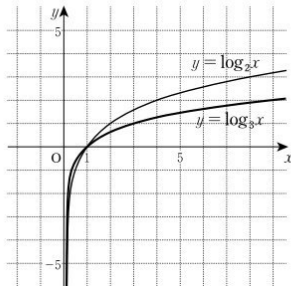
0

above

15

(2)

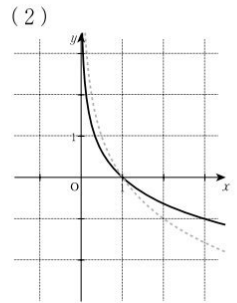
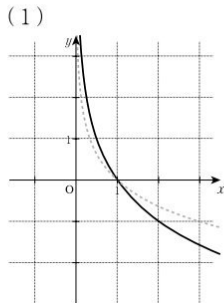
x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2



y

16

3.



4.

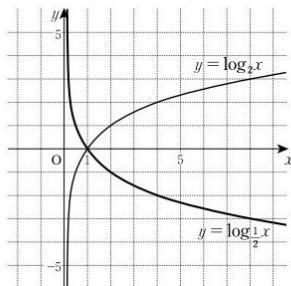
above

1

below

(3)

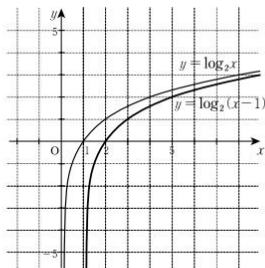
x	y
$\frac{1}{8}$	3
$\frac{1}{4}$	2
$\frac{1}{2}$	1
1	0
2	-1
4	-2
8	-3



y

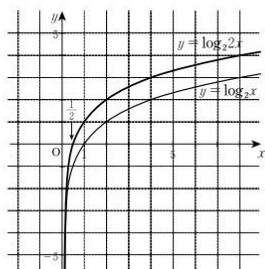
17

(1)



1

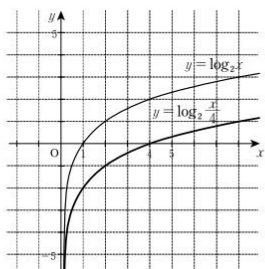
(2)



1

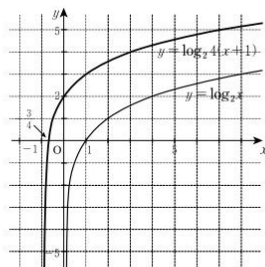
17

(3)



-2

(4)



-1

2

18

1.

(1) 3

(2) -4 units along the x -axis

(3) 2

(4) -3 units along the y -axis

(5) 3

1

18

2.

(1) $y = \log_3(x-1)$

(2) $y = \log_3(x+3)$

(3) $y = \log_3 x + 1$

(4) $y = \log_3 x - 2$

(5) $y = \log_3(x-2) - 3$

19	20
<p>1.</p> <p>(1) $\log_3 2 < \log_3 5$</p> <p>(2) $\log_2 7 < 2 \log_2 3$</p> <p>(3) $\log_{\frac{1}{2}} 5 < \log_{\frac{1}{2}} 3$</p>	<p>1.</p> <p>(1) (B) (4) (F)</p> <p>(2) (E) (5) (A)</p> <p>(3) (C) (6) (D)</p>
19	20
<p>2.</p> <p>(1) $3 \log_4 3 < 2 \log_2 3$</p> <p>(2) $\log_{\frac{1}{2}} 5 < \log_2 3$</p>	<p>2.</p> <p>(1) 1 unit along the y-axis</p> <p>(2) 3 units along the x-axis, and -2 units along the y-axis</p> <p>3.</p> <p>$\log_9 12 < 2 \log_3 2$</p>

L 21-24

21	22
<p>1.</p> <p>(3) $x = -\frac{1}{2}$</p> <p>(1) $x = 1$ (4) $x = \frac{7}{4}$</p> <p>(2) $x = 3$ (5) $x = -1$</p>	<p>(1) $x = 2$ (2) $x = 3$</p>
21	22
<p>2.</p> <p>(1) $x = 2$</p>	<p>(3) $x = 4$ (4) $x = 1$</p>

23	24
(1) $x = 27, 9$	1. (1) $2 < x < 4$ (2) $\frac{3}{2} < x < \frac{5}{3}$
23	24
(2) $x = 8, \frac{1}{2}$ (3) $x = \sqrt{2}, \frac{1}{8}$	2. (1) $-\frac{3}{2} < x < -1$ (2) $x > 14$

L 25–28

25

(1) $4 < x < 5$

25

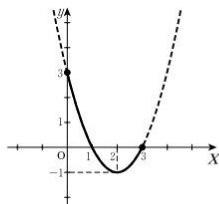
(2) $x > 3$

(3) $2 < x < 5$

26

(1) The maximum value is 3,
when $x = 1$.

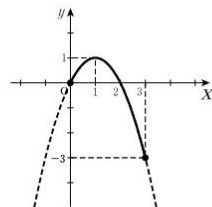
The minimum value is -1 ,
when $x = 4$.



26

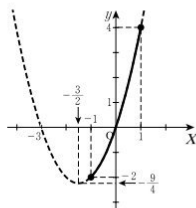
(2) The maximum value is 1,
when $x = 2$.

The minimum value is -3 ,
when $x = 8$.



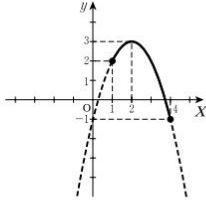
(3) The maximum value is 4,
when $x = 3$.

The minimum value is -2 ,
when $x = \frac{1}{3}$.



27	28
<p>1.</p> <p>(3) 0.3404</p> <p>(1) 0.1072 (4) 0.3617</p> <p>(2) 0.2553 (5) 0.3892</p>	<p>1.</p> <p>(1) 3.7781</p> <p>(2) 4.0791</p> <p>(3) -0.5229</p> <p>(4) -1.2219</p> <p>(5) -2.9209</p>
27	28
<p>2.</p> <p>(1) 3.3324 (3) 1.1987</p> <p>(2) 2.2648 (4) -1.6819</p>	<p>2.</p> <p>(1) 10-digit number</p> <p>(2) 16-digit number</p>

L 29–32

29	30
<p>1.</p> <p>(1) 16th decimal place</p>	<p>1.</p> <p>(1) $x = \frac{5}{2}$ (2) $5 < x < 6$</p>
29	30
<p>2.</p> <p>(1) $n = 13$</p> <p>(2) $n = 10$</p>	<p>2.</p> <p>The maximum value is 3, when $x = 4$.</p> <p>The minimum value is -1, when $x = 16$.</p> 

31

1.

(1) 5 (5) 1

(2) 5 (6) $\frac{2}{3}$

(3) $\frac{1}{2}$ (7) 10

(4) $\frac{1}{2}$ (8) $\frac{3}{4}$

2.

x	-1	0	1	2	3	4	5
$x-2$	-3	-2	-1	0	1	2	3
$ x-2 $	3	2	1	0	1	2	3

32

1.

x	-2	-1	0	1	2	3	4	5
$x(x-3)$	10	4	0	-2	-2	0	4	10
$ x(x-3) $	10	4	0	2	2	0	4	10

2.

x	-5	-4	-3	-2	-1	0	1	2	3
$(x+3)(x-1)$	12	5	0	-3	-4	-3	0	5	12
$ (x+3)(x-1) $	12	5	0	3	4	3	0	5	12

31

3.

(1)

(2)

4.

(1) $\begin{cases} x-1 & (\text{when } x \geq 1) \\ -(x-1) & (\text{when } x < 1) \end{cases}$

(2) $\begin{cases} x+3 & (\text{when } x \geq -3) \\ -(x+3) & (\text{when } x < -3) \end{cases}$

(3) $\begin{cases} x & (\text{when } x \geq 0) \\ -x & (\text{when } x < 0) \end{cases}$

(4) $\begin{cases} 2x-3 & (\text{when } x \geq \frac{3}{2}) \\ -(2x-3) & (\text{when } x < \frac{3}{2}) \end{cases}$

32

3.

(1) $\begin{cases} (x+1)(x-2) & (\text{when } x \leq -1 \text{ or } x \geq 2) \\ -(x+1)(x-2) & (\text{when } -1 < x < 2) \end{cases}$

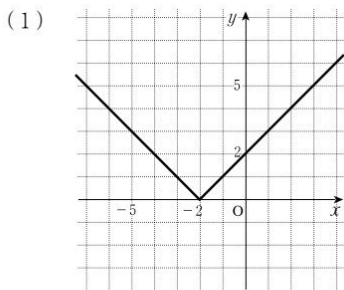
(2) $\begin{cases} x(x+3) & (\text{when } x \leq -3 \text{ or } x \geq 0) \\ -x(x+3) & (\text{when } -3 < x < 0) \end{cases}$

(3) $\begin{cases} (x-1)(x-3) & (\text{when } x \leq 1 \text{ or } x \geq 3) \\ -(x-1)(x-3) & (\text{when } 1 < x < 3) \end{cases}$

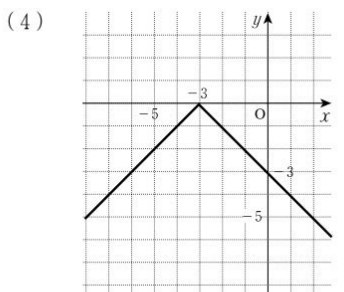
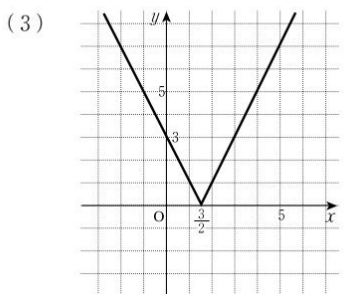
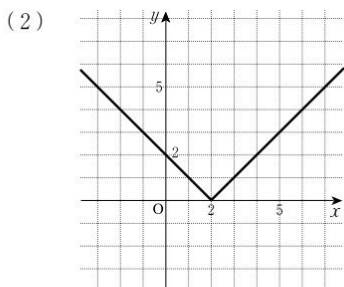
(4) $\begin{cases} x^2-9 & (\text{when } x \leq -3 \text{ or } x \geq 3) \\ -(x^2-9) & (\text{when } -3 < x < 3) \end{cases}$

(5) $\begin{cases} x^2-4x & (\text{when } x \leq 0 \text{ or } x \geq 4) \\ -(x^2-4x) & (\text{when } 0 < x < 4) \end{cases}$

33

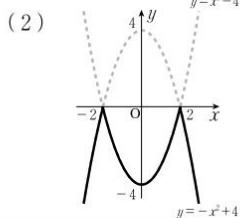
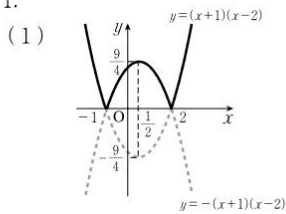


33



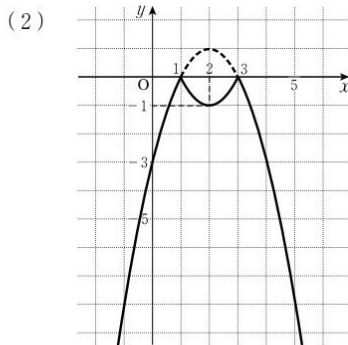
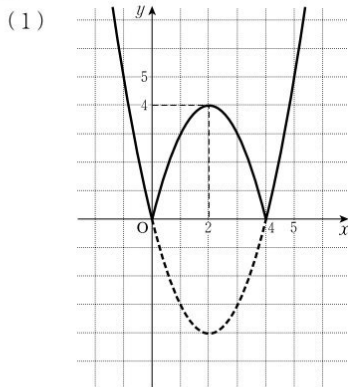
34

1.



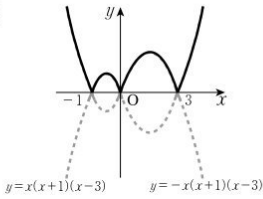
34

2.

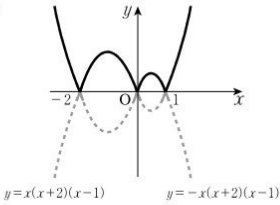


35

(1)

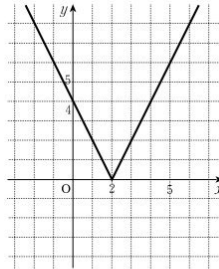


(2)

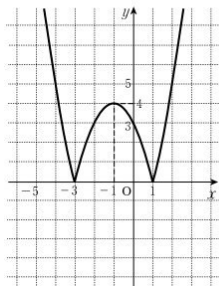


36

(1)

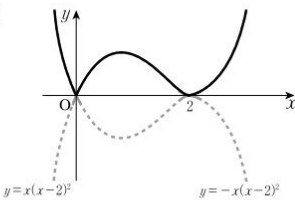


(2)

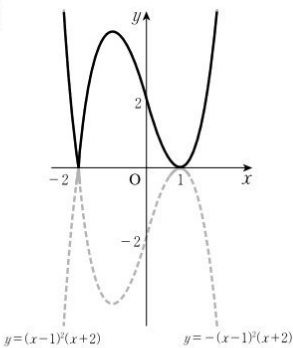


35

(3)

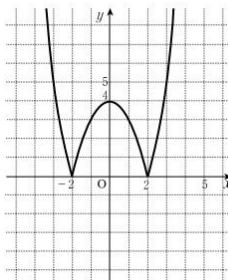


(4)

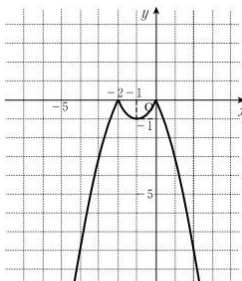


36

(3)



(4)



L 37-40

37

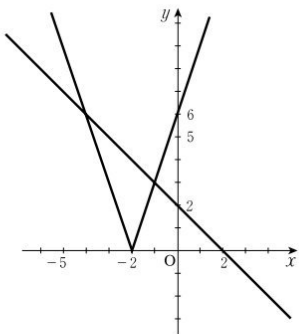
Ex.

38

Ex.

37

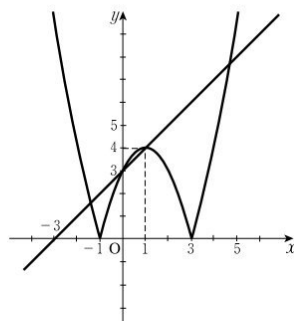
(1)



$(-1, 3), (-4, 6)$

38

(1)



$\left(\frac{3 \pm \sqrt{33}}{2}, \frac{9 \pm \sqrt{33}}{2}\right), (0, 3), (1, 4)$

39

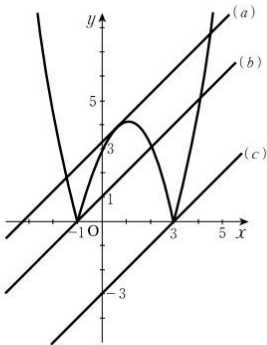
Ex.

40

1. (1) (B)
- (2) (F)
- (3) (C)
- (4) (E)
- (5) (A)
- (6) (D)

39

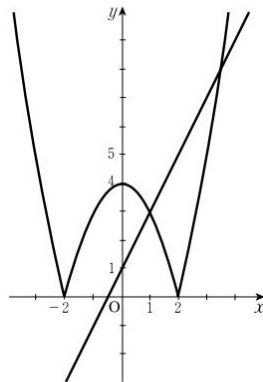
1.



$$-3 < k < 1, k > \frac{13}{4}$$

40

2.



$$(1 + \sqrt{6}, 3 + 2\sqrt{6}), (1, 3)$$

L 41-44

41	42
<p>1.</p> <p>(1) 4</p> <p>(2) 3</p> <p>(3) $\frac{3}{2}$</p> <p>(4) -1</p> <p>(5) $2a$</p>	<p>(1) -5</p> <p>(2) 6</p> <p>(3) 3</p> <p>(4) $\frac{1}{6}$</p> <p>(5) $\frac{2}{3}$</p>
41	42
<p>2.</p> <p>(1) 6</p> <p>(2) 2</p> <p>(3) 0</p> <p>(4) 1</p> <p>(5) 2</p>	<p>(6) 2</p> <p>(7) 3</p> <p>(8) $2a$</p> <p>(9) $2x$</p> <p>(10) $3x^2$</p>

43

1.

(1) 3

(2) $\frac{5}{2}$

44

1.

(1) -4

43

2.

(1)

(2)

(3)

(4)

(5)

44

2.

(1) -6

(2) 10

L 45–48

45	46
<p>1.</p> <p>(1) 6</p> <p>(2) 0</p> <p>(3) -2</p>	<p>1.</p> <p>(1) $f'(x) = 4x$</p>
45	46
<p>2.</p> <p>(1) 6</p> <p>(2) 0</p> <p>(3) -2</p> <p>(4) -6</p> <p>3.</p> <p>(i) <input type="text" value="2"/></p> <p>(ii) <input type="text" value="3"/></p>	<p>(2) $f'(x) = 2x$</p> <p>(3) $f'(x) = 2x + 2$</p>

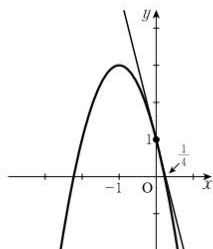
47	48
<p>(1) $f'(x) = 2$</p> <p>(2) $f'(x) = 1$</p> <p>(3) 0</p>	<p>1.</p> <p>(1) $y' = 15x^2$ (5) $y' = -8x$</p> <p>(2) $y' = -6x^2$ (6) $y' = -3$</p> <p>(3) $y' = 28x^3$ (7) $y' = 0$</p> <p>(4) $y' = 5x^4$ (8) $y' = 2ax$</p>
47	48
<p>(4) $f'(x) = 3x^2$</p> <p>(5) $4x^3$</p>	<p>2.</p> <p>(1) $y' = 6x - 5$</p> <p>(2) $y' = 4x - 5$</p> <p>(3) $y' = 8x^3 - 9x^2 + 4x$</p> <p>(4) $y' = -3 + 8x - 10x^4$ $(y' = -10x^4 + 8x - 3)$</p> <p>(5) $y' = 3x^3 + 2x$</p> <p>(6) $y' = 2ax + b$</p>

L 49–52

49	50
<p>1.</p> <p>(1) $y' = 8x + 12$</p> <p>(2) $y' = 16x^3$</p> <p>(3) $y' = 3x^2 + 12x + 12$</p> <p>(4) $y' = 8x^3 - 6x + 5$</p> <p>(5) $y' = 5x^4 + 3x^2 + 2x$</p>	<p>1.</p> <p>(1) $y' = 6x^2 - 4x - 4$</p> <p>(2) $y' = 45x^4 + 24x^3 + 9x^2 + 4x$</p> <p>(3)* $y' = 3x^2 + 12x + 12$</p>
49	50
<p>2.</p> <p>(1) $y' = 5x^4 + 2x$</p> <p>(2) $y' = 3x^2 - 2x - 2$</p> <p>(3) $y' = 24x^3 + 2x$</p>	<p>2.</p> <p>(1) $y' = 3ax^2 + 2bx + c$</p> <p>(2) $y' = 3x^2 - 4ax + a^2$</p> <p>(3) $y' = 4x^3 - 3x^2 - 8x + 4$</p> <p>(4) $y' = 3x^2 - 8x - 7$</p>

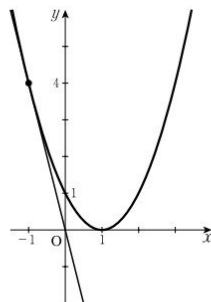
51

(1) $y = -4x + 1$



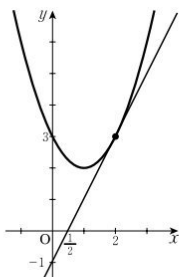
52

(1) $y = -4x$

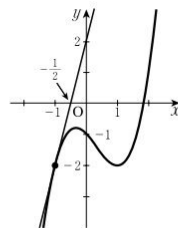


51

(2) $y = 2x - 1$

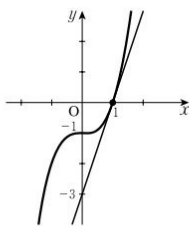


(2) $y = 4x + 2$

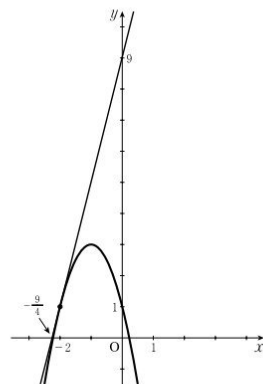


52

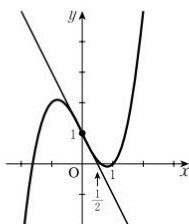
(3) $y = 3x - 3$



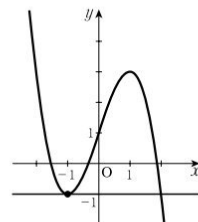
(3) $y = 4x + 9$



(4) $y = -2x + 1$



(4) $y = -1$



L 53–56

53	54
<p>1.</p> <p>(1) $y = x$</p> <p>(2) $y = 3$</p>	<p>1.</p> <p>$(3, 1), \left(-1, -\frac{1}{3}\right)$</p>
53	54
<p>2.</p> <p>$(1, 2)$</p> <p>$y = 4x - 2$</p>	<p>2.</p> <p>(1) $(1, 0), (-1, 2)$</p> <p>(2) When the point of contact is $(1, 0)$, $y = x - 1$</p> <p>When the point of contact is $(-1, 2)$, $y = x + 3$</p>

55	56
<p>1.</p> $a = 3, b = -11, c = 9$	<p>1.</p> $a = -2, b = 3, c = -1$
55	56
<p>2.</p> $a = -2, b = 4, c = 1$	<p>2.</p> $a = -\frac{1}{3}, b = 1, c = 1, d = 1$

L 57–60

57	58
Ex.	Ex.
57	58
1. $y = x - 3$ $(-1, -4)$	1. $y = 7x$ $y = -x$

59	60
<p>1.</p> $y = -2x + 1$ $y = \frac{19}{4}x + \frac{31}{4}$	<p>1.</p> $y = 7x - 1$ <p>2.</p> $a = 2, b = -3$
59	60
<p>2.</p> $y = 7x + 5$	<p>3.</p> $y = -3x$ $y = \frac{15}{4}x$

L 61-64

61

1. (i) increasing

(ii) relative maximum

(iii) =

61

(i) decreasing

(ii) <

(iii) relative minimum

(iv) =

(v) increasing

(vi) >

2. (i) increasing

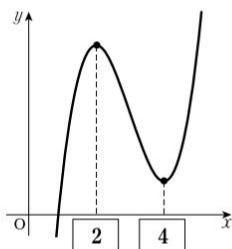
(ii) decreasing

(iii) relative maximum

relative minimum

62

1. $3(x-4)(x-2)$



2

5

4

1

62

2. 1 0 -1

1

x	...	-1	...	1	...
y'	+	0	-	0	+
y	↗	3	↘	-1	↗
		(relative maximum)		(relative minimum)	

-1

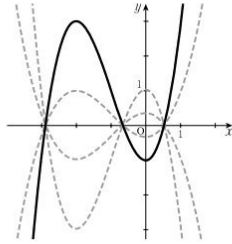
1

63

1.

x	...	-2	...	0	...
y'	+	0	-	0	+
y	↗	3	↘	-1	↗

3	-2
-1	0

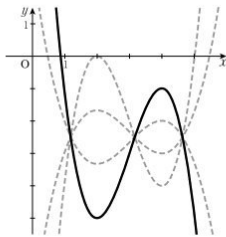


63

2.

x	...	2	...	4	...
y'	-	0	+	0	-
y	↘	-5	↗	-1	↘

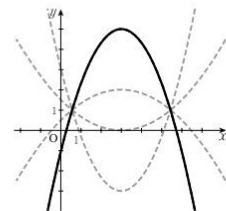
-1	4
-5	2



3.

x	...	3	...
y'	+	0	-
y	↗	5	↘

5	3
---	---

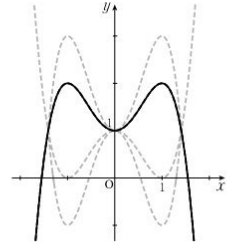


64

1.

x	...	-1	...	0	...	1	...
y'	+	0	-	0	+	0	-
y	↗	2	↘	1	↗	2	↘

2	-1
1	0
2	1

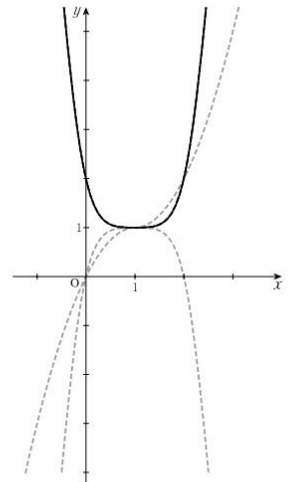


64

2.

x	...	1	...
y'	-	0	+
y	↘	1	↗

1	1
---	---



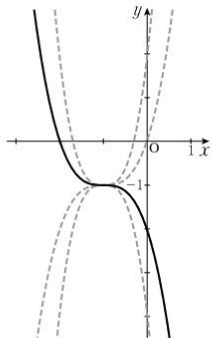
L 65–68

65

1.

x	...	-1	...
y'	-	0	-
y	↘	-1	↘

There are no relative extreme values.



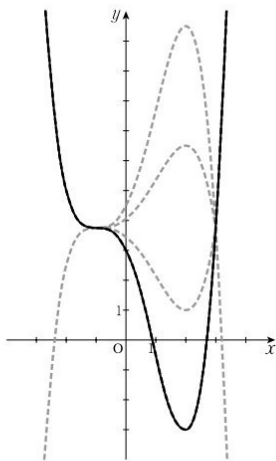
65

2.

x	...	-1	...	2	...
y'	-	0	-	0	+
y	↘	$\frac{15}{4}$	↘	-3	↗

There is no relative maximum value.

The relative minimum value is -3 , at $x = 2$.



66

1.

$$(1) y' = -(3x+1)(x-1)$$

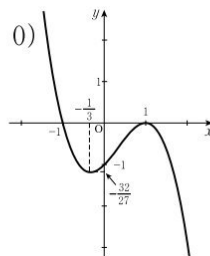
x	...	$-\frac{1}{3}$...	1	...
y'	-	0	+	0	-
y	↘	$-\frac{32}{27}$	↗	0	↘

0	1
---	---

$-\frac{32}{27}$	$-\frac{1}{3}$
------------------	----------------

$$(2) \textcircled{1} (1, 0), (-1, 0)$$

$$\textcircled{2} (0, -1)$$



66

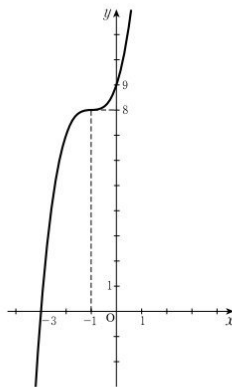
2.

x	...	-1	...
y'	+	0	+
y	↗	8	↗

There are no relative extreme values.

$$(2) \textcircled{1} (-3, 0)$$

$$\textcircled{2} (0, 9)$$



67

1.

x	...	-1	...	0	...	1	...
y'	-	0	+	0	-	0	+
y	↘	0	↗	1	↘	0	↗

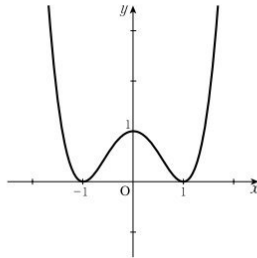
The relative minimum value is 0, at $x = -1$.

The relative maximum value is 1, at $x = 0$.

The relative minimum value is 0, at $x = 1$.

(2) ① $(-1, 0), (1, 0)$

② $(0, 1)$



68

1.

There are no relative extreme values.

2.

There are no relative extreme values.

67

2.

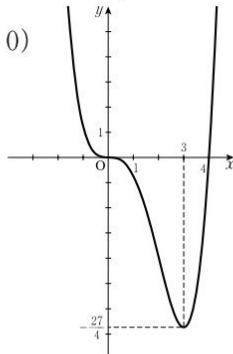
x	...	0	...	3	...
y'	-	0	-	0	+
y	↘	0	↘	$-\frac{27}{4}$	↗

There is no relative maximum value.

The relative minimum value is $-\frac{27}{4}$, at $x = 3$.

(2) ① $(0, 0), (4, 0)$

② $(0, 0)$



68

3.

$$\frac{D}{4} = (a+3)(a-3) \leq 0$$

$$-3 \leq a \leq 3$$

4.

$$\frac{D}{4} = a(a-9) > 0$$

$$a < 0, a > 9$$

L 69–72

69

1.
 (1) (A) (3) (C)
 (2) (D) (4) (B)

69

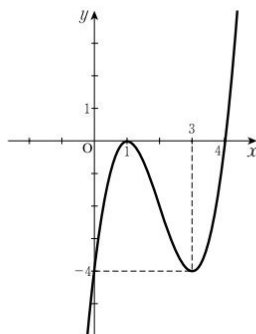
2.
 (1) (F) (3) (B)
 (2) (C) (4) (D)

70

1.
 (1)
- | | | | | | |
|------|-----|---|-----|----|-----|
| x | ... | 1 | ... | 3 | ... |
| y' | + | 0 | - | 0 | + |
| y | ↗ | 0 | ↘ | -4 | ↗ |

The relative maximum value is 0, at $x = 1$.
 The relative minimum value is -4 , at $x = 3$.

- (2) ① (1, 0), (4, 0)
 ② (0, -4)



70

2.
 $-2 \leq a \leq 3$

71

72

Ex.

1.

$$a = 1, b = 1$$

At $x = -\frac{1}{3}$, there is a relative
minimum value of $\frac{49}{27}$.

71

72

1.

$$(1) a = -3, b = -9$$

(2)

x	...	-1	...	3	...
$f'(x)$	+	0	-	0	+
$f(x)$	↗	17	↘	-15	↗

At $x = -1$, there is a relative
maximum value of 17.

2.

$$a = -3, b = 0, c = 4$$

At $x = 0$, there is a relative
maximum value of 4.

L 73–76

73	74																		
<p>1.</p> <p>(1)</p> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">\dots</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">\dots</td> <td style="padding: 5px;">$-\frac{1}{3}$</td> <td style="padding: 5px;">\dots</td> </tr> <tr> <td style="padding: 5px;">$f'(x)$</td> <td style="padding: 5px;">$-$</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">$+$</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">$-$</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td style="padding: 5px;">\searrow</td> <td style="padding: 5px;">relative minimum</td> <td style="padding: 5px;">\nearrow</td> <td style="padding: 5px;">relative maximum</td> <td style="padding: 5px;">\searrow</td> </tr> </table> <p style="margin-left: 20px;">The relative maximum value is $-\frac{4}{27}a + 1$, at $x = -\frac{1}{3}$.</p>	x	\dots	-1	\dots	$-\frac{1}{3}$	\dots	$f'(x)$	$-$	0	$+$	0	$-$	$f(x)$	\searrow	relative minimum	\nearrow	relative maximum	\searrow	<p>1.</p> <div style="font-size: 2em; margin-left: 10px;">{</div> <p style="margin-left: 20px;">When $a > 0$, the relative minimum value is $-2a + 4$, at $x = 1$.</p> <p style="margin-left: 20px;">When $a < 0$, the relative minimum value is $2a + 4$, at $x = -1$.</p>
x	\dots	-1	\dots	$-\frac{1}{3}$	\dots														
$f'(x)$	$-$	0	$+$	0	$-$														
$f(x)$	\searrow	relative minimum	\nearrow	relative maximum	\searrow														
73	74																		
<p>2.</p> <p>(1) The relative maximum value is $27a + 1$, at $x = -3$.</p> <p>(2) The relative maximum value is $-5a + 1$, at $x = 1$.</p>	<p>2.</p> <div style="font-size: 2em; margin-left: 10px;">{</div> <p style="margin-left: 20px;">When $a > 0$, the relative maximum value is $\frac{40}{27}a + 8$, at $x = -\frac{2}{3}$.</p> <p style="margin-left: 20px;">When $a < 0$, the relative maximum value is $-8a + 8$, at $x = 2$.</p>																		

75	76
<p>1.</p> <p>(1) The relative minimum value is -81, at $x = -3$.</p>	<p>Ex.</p>
75	76
<p>2.</p> <p>The relative minimum value is -24, at $x = -2$.</p>	<p>1.</p> <p> $\left\{ \begin{array}{l} \text{When } a > 0, \text{ the relative maximum} \\ \text{value is } \frac{4}{27}a^3, \text{ at } x = \frac{1}{3}a. \\ \text{When } a = 0, \text{ there is no relative} \\ \text{maximum value.} \\ \text{When } a < 0, \text{ the relative maximum} \\ \text{value is } 0, \text{ at } x = a. \end{array} \right.$ </p>

L 77–80

77

Ex.

78

Ex.

77

1.

$$a = 1$$

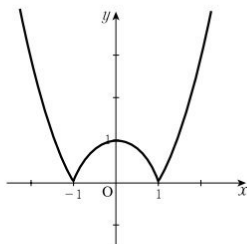
78

(1)

x	...	-1	...	0	...	1	...
y'	-	X	+	0	-	X	+
y	↘	0	↗	1	↘	0	↗

The relative minimum value is 0,
at $x = -1, 1$.

The relative maximum value is 1,
at $x = 0$.



79

80

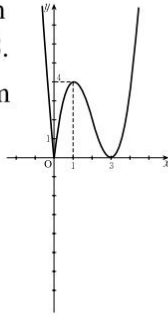
(1) 0

$$y = x^3 - 6x^2 + 9x$$

x	...	0	...	1	...	3	...
y'	-	0	+	0	-	0	+
y	↘	0	↗	4	↘	0	↗

The relative minimum value is 0, at $x = 0, 3$.

The relative maximum value is 4, at $x = 1$.



1.

$\left\{ \begin{array}{l} \text{When } a > 0, \text{ the relative maximum value is } 7a + 3, \text{ at } x = -1. \\ \text{When } a < 0, \text{ the relative maximum value is } -20a + 3, \text{ at } x = 2. \end{array} \right.$

79

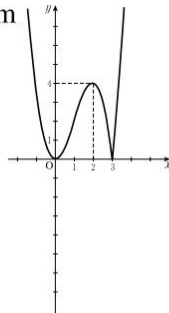
80

(2)

x	...	0	...	2	...	3	...
y'	-	0	+	0	-	0	+
y	↘	0	↗	4	↘	0	↗

The relative minimum value is 0, at $x = 0, 3$.

The relative maximum value is 4, at $x = 2$.



2.

$\left\{ \begin{array}{l} \text{When } a > 0, \text{ the relative minimum value is } -a^3 + 1, \text{ at } x = a. \\ \text{When } a = 0, \text{ there is no relative minimum value.} \\ \text{When } a < 0, \text{ the relative minimum value is } 1, \text{ at } x = 0. \end{array} \right.$

L 81–84

81

1.

②

1	0
---	---

-1	1
----	---

③

3	-1
---	----

-17	-3
-----	----

82

1.

(1) The maximum value is 4, at $x = 1$.
The minimum value is 0, at $x = 0$.

(2) The maximum value is 4,
at $x = 1, 4$.

The minimum value is 0,
at $x = 0, 3$.

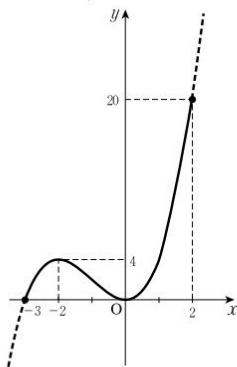
81

2.

x	-3	...	-2	...	0	...	2
y'	+	+	0	-	0	+	+
y	0	↗	4	↘	0	↗	20

The maximum value is 20, at $x = 2$.

The minimum value is 0,
at $x = -3, 0$.



82

2.

(1) The maximum value is 0,
at $x = 0, 2$.
The minimum value is -225 ,
at $x = -3$.

(2) The maximum value is 0,
at $x = 0, 2$.

The minimum value is -9 ,
at $x = 3$.

83

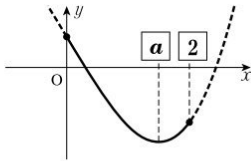
Ex.

83

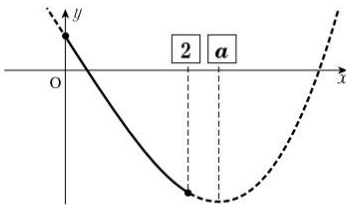
1.

x	...	$-a$...	a	...
$f'(x)$	+	0	-	0	+
$f(x)$	↗	relative maximum	↘	relative minimum	↗

- (i) When $0 < a \leq 2$,
 $f(a) = -a^3$



- (ii) When $a > 2$,
 $f(2) = 8 - 6a^2 + a^3$



84

Ex.

84

1.

x	...	$-a$...	a	...
$f'(x)$	+	0	-	0	+
$f(x)$	↗	relative maximum	↘	relative minimum	↗

- (i) When $0 < a \leq \frac{2\sqrt{3}}{3}$,

$$f(2) = 8 - 6a^2 + a^3$$

- (ii) When $a > \frac{2\sqrt{3}}{3}$,

$$f(0) = a^3$$

L 85–88

85	86
<p>1.</p> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-bottom: 5px;">$0 < a \leq 2$</div> <div style="display: flex; gap: 10px; margin-top: 5px;"> <div style="border: 1px solid black; padding: 2px;">a^3</div> <div style="border: 1px solid black; padding: 2px;">a</div> </div> <div style="margin-top: 10px;"><div style="border: 1px solid black; padding: 2px;">$a > 2$</div></div> <div style="margin-top: 10px; display: flex; gap: 10px;"> <div style="border: 1px solid black; padding: 2px;">$2a^3 - 12a + 16$</div> <div style="border: 1px solid black; padding: 2px;">2</div> </div>	<p>Ex.</p>
85	86
<p>2.</p> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-bottom: 5px;">$0 < a \leq \frac{4}{3}$</div> <div style="display: flex; gap: 10px; margin-top: 5px;"> <div style="border: 1px solid black; padding: 2px;">$2a^3 - 12a + 16$</div> <div style="border: 1px solid black; padding: 2px;">2</div> </div> <div style="margin-top: 10px;"><div style="border: 1px solid black; padding: 2px;">$a > \frac{4}{3}$</div></div> <div style="margin-top: 10px; display: flex; gap: 10px;"> <div style="border: 1px solid black; padding: 2px;">$2a^3$</div> <div style="border: 1px solid black; padding: 2px;">0</div> </div>	<p>1.</p> <div style="font-size: 2em; vertical-align: middle; margin-right: 5px;">{</div> <p style="margin-left: 10px;">When $a > 0$, the minimum value is $-4a + 3$, at $x = -1$.</p> <p style="margin-left: 10px;">When $a = 0$, the minimum value is 3.</p> <p style="margin-left: 10px;">When $a < 0$, the minimum value is 3, at $x = 0$.</p>

87	88
<p>1.</p> <p> $\left\{ \begin{array}{l} \text{When } 0 < a \leq \frac{4}{3}, \text{ the minimum value} \\ \text{is } -16a + 12a^2, \text{ at } x = 2. \\ \text{When } a > \frac{4}{3}, \text{ the minimum value is} \\ 0, \text{ at } x = 0. \end{array} \right.$ </p>	<p>Ex.</p>
87	88
<p>2.</p> <p>The minimum value is 0, at $x = 0$.</p>	<p>1.</p> <p> $\left\{ \begin{array}{l} \text{When } 0 < a < \frac{1}{2} \text{ and when } a > 2, \\ \text{the maximum value is} \\ -2a^3 + 18a^2 - 24a + 8, \text{ at } x = 2. \\ \text{When } \frac{1}{2} \leq a \leq 2, \text{ the maximum} \\ \text{value is } 2a^3, \text{ at } x = a. \end{array} \right.$ </p>

L 89–92

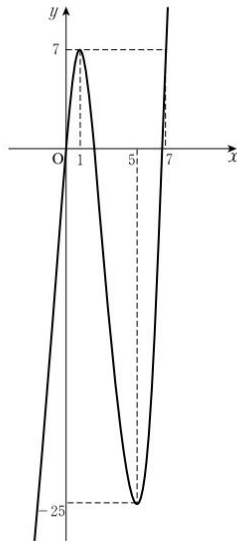
89	90
<p>Ex.</p>	<p>1.</p> $\left\{ \begin{array}{l} \text{When } 0 < a \leq \frac{4}{3}, \text{ the minimum value} \\ \text{is } -8 + 6a, \text{ at } x = 2. \\ \text{When } a > \frac{4}{3}, \text{ the minimum value is} \\ 0, \text{ at } x = 0. \end{array} \right.$
89	90
<p>1.</p> $a = -\frac{9}{10}, b = 10$	<p>2.</p> <p>The maximum value is $-20a + 3$, at $x = 2$.</p>

91

Ex.

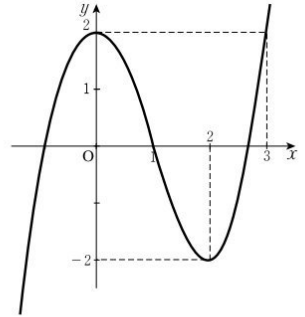
91

1.
 - (i) When $0 < a < 1$,
 $f(a) = a^3 - 9a^2 + 15a$
 - (ii) When $1 \leq a \leq 7$,
 $f(1) = 7$
 - (iii) When $a > 7$,
 $f(a) = a^3 - 9a^2 + 15a$



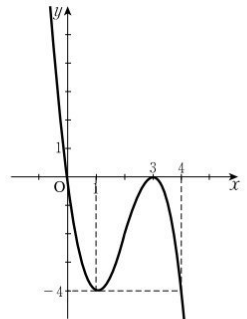
92

1.
 - (i) When $0 < a \leq 3$,
 $f(0) = 2$
 - (ii) When $a > 3$,
 $f(a) = a^3 - 3a^2 + 2$



92

2.
 - (i) When $0 < a < 1$,
 $f(a) = -a^3 + 6a^2 - 9a$
 - (ii) When $1 \leq a \leq 4$,
 $f(1) = -4$
 - (iii) When $a > 4$,
 $f(a) = -a^3 + 6a^2 - 9a$



93

1.
(i) When $0 < a < \frac{1}{2}$,

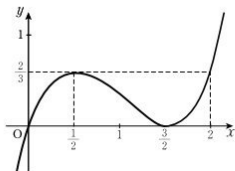
$$f(a) = \frac{4}{3}a^3 - 4a^2 + 3a$$

- (ii) When $\frac{1}{2} \leq a \leq 2$,

$$f\left(\frac{1}{2}\right) = \frac{2}{3}$$

- (iii) When $a > 2$,

$$f(a) = \frac{4}{3}a^3 - 4a^2 + 3a$$



93

- 2.*
(i) When $0 < a < 1$,

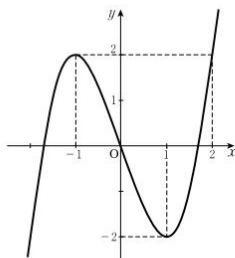
$$f(-a) = -a^3 + 3a$$

- (ii) When $1 \leq a \leq 2$,

$$f(-1) = 2$$

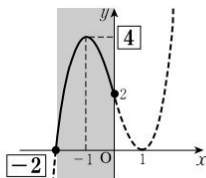
- (iii) When $a > 2$,

$$f(a) = a^3 - 3a$$



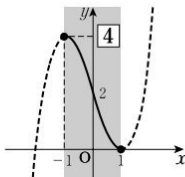
94

1.
(1) $-2 \leq x \leq 0$
 $f(-1) = 4$



The relative maximum value

- (2) $-1 \leq x \leq 1$
 $f(-1) = 4$



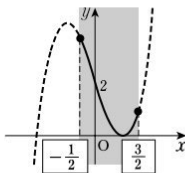
The left end of the domain

The relative maximum value

Note: In this case both of the above apply.

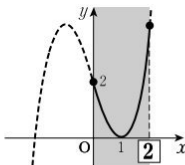
94

- (3) $-\frac{1}{2} \leq x \leq \frac{3}{2}$
 $f\left(-\frac{1}{2}\right) = \frac{27}{8}$



The left end of the domain

- (4) $0 \leq x \leq 2$
 $f(2) = 4$



The right end of the domain

2.

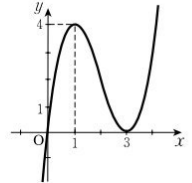
$$a = \frac{-3 + \sqrt{6}}{3}$$

95

Ex.

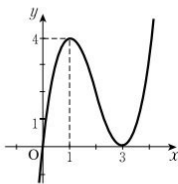
96

- 1.
- (i) When $a < \frac{3-\sqrt{6}}{3}$,
 $f(a) = a^3 - 6a^2 + 9a$
- (ii) When $\frac{3-\sqrt{6}}{3} \leq a < 1$,
 $f(a+2) = a^3 - 3a + 2$
- (iii) When $1 \leq a \leq 3$,
 $f(3) = 0$
- (iv) When $a > 3$,
 $f(a) = a^3 - 6a^2 + 9a$



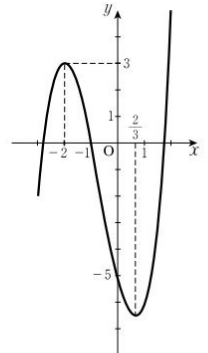
95

- 1.
- (i) When $a < -1$,
 $f(a+2) = a^3 - 3a + 2$
- (ii) When $-1 \leq a \leq 1$,
 $f(1) = 4$
- (iii) When $1 < a < \frac{3+\sqrt{6}}{3}$,
 $f(a) = a^3 - 6a^2 + 9a$
- (iv) When $a \geq \frac{3+\sqrt{6}}{3}$,
 $f(a+2) = a^3 - 3a + 2$



96

- 2.
- (i) When $t < -2$,
 $f(t) = t^3 + 2t^2 - 4t - 5$
- (ii) When $-2 \leq t \leq 0$,
 $f(-2) = 3$
- (iii) When $0 < t < \frac{1+\sqrt{13}}{3}$,
 $f(t-2) = t^3 - 4t^2 + 3$
- (iv) When $t \geq \frac{1+\sqrt{13}}{3}$,
 $f(t) = t^3 + 2t^2 - 4t - 5$



L 97–100

97

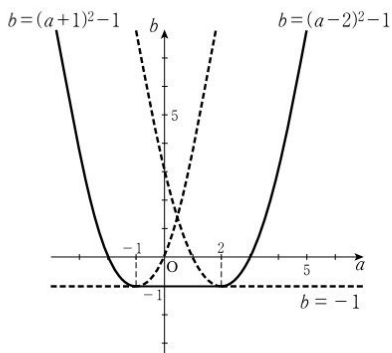
Ex.

98

Ex.

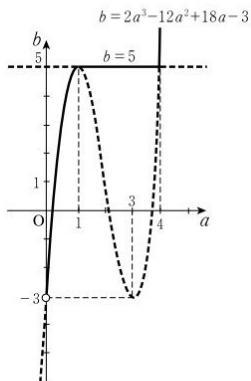
97

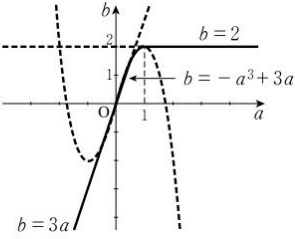
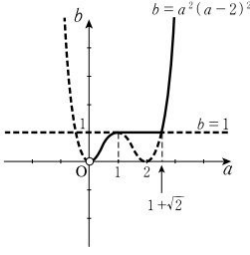
- 1.
- (i) When $a < -1$,
 $b = (a+1)^2 - 1$
 - (ii) When $-1 \leq a \leq 2$,
 $b = -1$
 - (iii) When $a > 2$,
 $b = (a-2)^2 - 1$



98

- 1.
- (i) When $0 < a < 1$,
 $b = 2a^3 - 12a^2 + 18a - 3$
 - (ii) When $1 \leq a \leq 4$,
 $b = 5$
 - (iii) When $a > 4$,
 $b = 2a^3 - 12a^2 + 18a - 3$



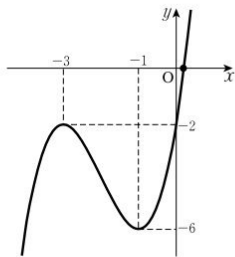
99	100
<p>Ex.</p>	<p>1.</p> <p>(i) When $0 < a < 1$, $f(-a) = -a^3 + 3a + 2$</p> <p>(ii) When $1 \leq a \leq 2$, $f(-1) = 4$</p> <p>(iii) When $a > 2$, $f(a) = a^3 - 3a + 2$</p>
99	100
<p>1.</p> <p>(i) When $a < 0$, $b = 3a$</p> <p>(ii) When $a = 0$, $b = 0$</p> <p>(iii) When $0 < a \leq 1$, $b = -a^3 + 3a$</p> <p>(iv) When $a > 1$, $b = 2$</p> 	<p>2.</p> <p>(i) When $0 < a < 1$, $b = a^2(a-2)^2$</p> <p>(ii) When $1 \leq a \leq 1 + \sqrt{2}$, $b = 1$</p> <p>(iii) When $a > 1 + \sqrt{2}$, $b = a^2(a-2)^2$</p> 

L 101-104

101

1.

(1)

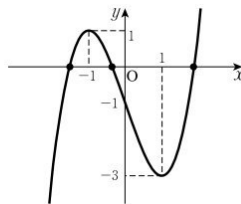


1 real root

102

1.

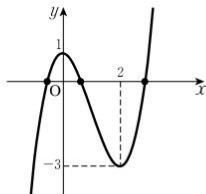
(1)



1 positive root and 2 negative roots

101

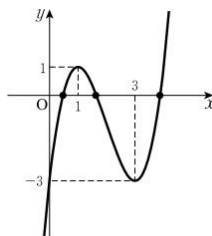
(2)



3 real roots

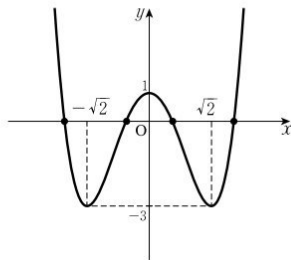
102

(2)



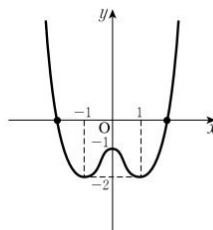
3 positive roots

(3)



4 real roots

(3)



1 positive root and 1 negative root

103

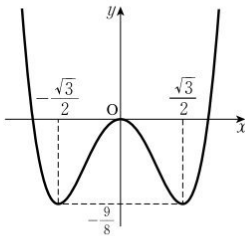
Ex.

104

Ex.

103

1.



When $a > 0$, there are 2 real roots.

When $a = 0$, there are 3 real roots.

When $-\frac{9}{8} < a < 0$,

there are 4 real roots.

When $a = -\frac{9}{8}$, there are 2 real roots.

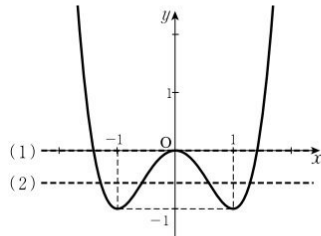
When $a < -\frac{9}{8}$, there are 0 real roots.

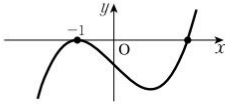
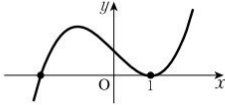
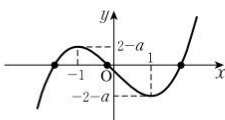
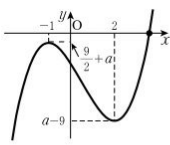
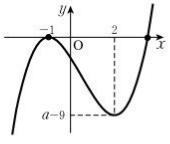
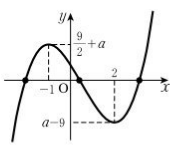
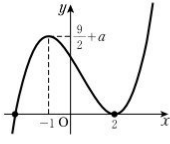
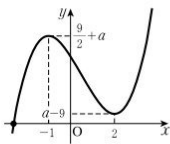
104

1.

(1) $a = 0$

(2) $-1 < a < 0$



105	106
<p>Ex.</p>	<p>1.</p> $-\sqrt{2} < a < 0, 0 < a < \sqrt{2}$
<p>105</p> <p>1.</p> <p>(1) $a = \pm 2$</p>  <p>(2) $-2 < a < 2$</p>  	<p>106</p> <p>2.</p> <p>(i) When $a < -\frac{9}{2}$,</p> <p>there is 1 point of intersection.</p>  <p>(ii) When $a = -\frac{9}{2}$,</p> <p>there are 2 points of intersection.</p>  <p>(iii) When $-\frac{9}{2} < a < 9$,</p> <p>there are 3 points of intersection.</p>  <p>(iv) When $a = 9$,</p> <p>there are 2 points of intersection.</p>  <p>(v) When $a > 9$,</p> <p>there is 1 point of intersection.</p> 

107

1.

Let $f(x) = x^4 + 28 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$

x	...	0	...	3	...
$f'(x)$	-	0	-	0	+
$f(x)$	↘	28	↘	1	↗

The minimum value of $f(x)$ is 1.

Therefore, $f(x) > 0$

Thus, $x^4 + 28 > 4x^3$

108

1.

Let $f(x) = \left(x + \frac{1}{2}\right)^3 - 3x^2 - \frac{1}{8}$

$f(x) = x^3 - \frac{3}{2}x^2 + \frac{3}{4}x$

$f'(x) = 3x^2 - 3x + \frac{3}{4}$

$= 3\left(x - \frac{1}{2}\right)^2 \geq 0$

Therefore, $f(x)$ is monotonically increasing.

As $f(0) = 0$,

$f(x) > 0$ for $x > 0$.

Thus,

when $x > 0$, $\left(x + \frac{1}{2}\right)^3 > 3x^2 + \frac{1}{8}$

107

2.

Let $f(x) = x^3 + 7x + 1 - 3x^2$

$f'(x) = 3x^2 - 6x + 7$

$= 3(x - 1)^2 + 4 > 0$

Therefore, $f(x)$ is monotonically increasing.

As $f(0) = 1$,

$f(x) > 1$ for $x > 0$.

Thus,

when $x > 0$, $x^3 + 7x + 1 > 3x^2$

108

2.

Let $f(x) = 2x^3 - 9x^2 + 12x + 1$

$f'(x) = 6x^2 - 18x + 12$

$= 6(x - 2)(x - 1)$

x	0	...	1	...	2	...
$f'(x)$	+	+	0	-	0	+
$f(x)$	1	↗	6	↘	5	↗

As $f(0) = 1$ and $f(2) = 5$,

$f(x) > 0$ for $x > 0$.

Thus,

when $x > 0$, $2x^3 - 9x^2 + 12x + 1 > 0$

L 109–112

109	110												
<p>1.</p> $p < -2, p > 2$	<p>1.</p> $-27 < m < 0$												
109	110												
<p>2.</p> $-\sqrt{2} < p < \sqrt{2}$	<p>2.</p> <p>Let $f(x) = \frac{1}{4}(x^4 - 1) + \frac{1}{2}(x^2 - 1) - \frac{2}{3}(x^3 - 1)$</p> $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{12}$ $f'(x) = x^3 - 2x^2 + x$ $= x(x^2 - 2x + 1) = x(x - 1)^2$ <table border="1" data-bbox="553 1134 826 1273"> <tbody> <tr> <td>x</td> <td>0</td> <td>...</td> <td>1</td> </tr> <tr> <td>$f'(x)$</td> <td>0</td> <td>+</td> <td>0</td> </tr> <tr> <td>$f(x)$</td> <td>$-\frac{1}{12}$</td> <td>↗</td> <td>0</td> </tr> </tbody> </table> <p>From the graph, when $0 \leq x \leq 1$,</p> $\frac{1}{4}(x^4 - 1) + \frac{1}{2}(x^2 - 1) \leq \frac{2}{3}(x^3 - 1)$	x	0	...	1	$f'(x)$	0	+	0	$f(x)$	$-\frac{1}{12}$	↗	0
x	0	...	1										
$f'(x)$	0	+	0										
$f(x)$	$-\frac{1}{12}$	↗	0										

111	112
<p>1.</p> <p>(1) x</p> <p>(2) x^3</p> <p>(3) $2x$</p> <p>2.</p> <p>(1) x^2</p> <p>(2) x^2</p> <p>(3) x^2</p> <p>3.</p> <p>x^2</p>	<p>1.</p> <p>(1) $\frac{3}{4}x^4 + C$ (5) $\frac{2}{3}t^3 + C$</p> <p>(2) $-\frac{2}{3}x^3 + C$ (6) $2t^4 + C$</p> <p>(3) $2x^3 + C$ (7) $-3x + C$</p> <p>(4) $\frac{a}{3}x^3 + C$ (8) $ax + C$</p>
111	112
<p>4.</p> <p>(1) $\frac{1}{4}x^4 + C$ (4) $x^2 + C$</p> <p>(2) $\frac{1}{5}x^5 + C$ (5) $x^3 + C$</p> <p>(3) $\frac{1}{6}x^6 + C$ (6) $x + C$</p>	<p>2.</p> <p>(1) $2x^2 + 3x + C$</p> <p>(2) $\frac{1}{3}x^3 + \frac{3}{2}x^2 + x + C$</p> <p>(3) $\frac{1}{3}t^3 - \frac{3}{2}t^2 + t + C$</p> <p>(4) $\frac{1}{3}x^3 + x^2 + ax + C$</p> <p>(5) $\frac{p}{4}x^4 + \frac{q}{3}x^3 + \frac{r}{2}x^2 + C$</p> <p>(6) $x^2 + a^2x + C$</p> <p>(7) $s^2t - \frac{1}{3}t^3 + C$</p> <p>(8) $\frac{1}{3}s^3 - t^2s + C$</p>

L 113–116

113	114
<p>(1) $\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + C$</p> <p>(2) $\frac{4}{3}x^3 + 2x^2 + x + C$</p> <p>(3) $\frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + C$</p> <p>(4) $\frac{1}{4}x^4 + x + C$</p> <p>(5) $\frac{1}{3}x^3 + \frac{a+b}{2}x^2 + abx + C$</p>	<p>1.</p> <p>(1) $F(x) = -\frac{1}{2}x^2 - 1$</p> <p>(2) $F(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + \frac{5}{4}$</p>
113	114
<p>(6) $\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x + C$</p> <p>(7) $\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + C$</p> <p>(8) $\frac{1}{4}t^4 + \frac{5}{3}t^3 + 4t^2 + 4t + C$</p> <p>(9) $2x^2 + C$</p>	<p>2.</p> <p>(1) $F(x) = -x^2 + 3x - 2$</p> <p>(2) $F(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - \frac{1}{2}$</p> <p>(3) $F(x) = x^3 + x + 4$</p>

115	116
1. $f(x) = -2x^2 + 8x - 5$	1. $f(x) = x^3 + 1$
115	116
2. $f(x) = \frac{1}{2}x^2 - x - \frac{1}{2}$	2. $f(x) = 2x^2 - 8x + 7$ 3. $f(x) = -x^3 + x^2 - x + 5$

L 117–120

117	118
<p>1.</p> <p>(1) $F(x) = x^2 + x + 1$ $F(2) - F(1) = 4$</p> <p>(2) 4</p> <p>(3) 4</p>	<p>(1) $\frac{1}{3}$</p> <p>(2) $-\frac{1}{3}$</p> <p>(3) $-\frac{1}{3}$</p> <p>(4) $\frac{2}{3}$</p> <p>(5) 0</p> <p>(6) $a^2 - 1$</p>
117	118
<p>2.</p> <p>(1) 10</p> <p>(2) -2</p> <p>(3) $\frac{3}{2}$</p>	<p>(7) $\frac{7}{3}$</p> <p>(8) $\frac{16}{3}$</p> <p>(9) $\frac{16}{3}$</p> <p>(10) $\frac{20}{3}$</p> <p>(11) 15</p>

119	120
<p>1.</p> <p>(1) 0</p> <p>(2) 20</p> <p>(3) $\frac{98}{3}$</p> <p>(4) $\frac{8}{3}$</p> <p>(5) $\frac{20}{3}$</p>	<p>1.</p> $x^3 + 2x^2 - 5$ <p>2.</p> $f(x) = 2x^3 - x^2 - 3x + 9$
119	120
<p>2.</p> <p>(1) $f(x) = 2x^2 + 2x - 1$</p>	<p>3.</p> <p>(1) 1</p> <p>(2) 9</p> <p>(3) $\frac{4}{3}a^3$</p>

L 121–124

121	122
<p>1.</p> <p>(1) (i) $\frac{3}{2}$</p> <p>(ii) $\frac{3}{2}$</p>	<p>(1) 12</p> <p>(2) $-\frac{9}{2}$</p> <p>(3) $-\frac{15}{4}$</p>
121	122
<p>2.</p> <p>(1) $\frac{15}{2}$</p> <p>(2) $-\frac{4}{3}$</p> <p>(3) -6</p>	<p>(4) $\frac{32}{3}$</p> <p>(5) 9</p> <p>(6) $-\frac{8}{3}$</p> <p>(7) 2</p>

123	124
<p>1.</p> <p>(1) 3</p> <p>(2) 6</p> <p>(3) $\frac{9}{2}$</p> <p>(4) 0</p> <p>(5) 9</p> <p>(6) 18</p> <p>(7) $\frac{81}{4}$</p> <p>(8) 0</p> <p>2.</p> <p>(i) <input type="text" value="0"/></p> <p>(ii) $2 \int_{\text{0}}^3 x^n dx$</p>	<p>1.</p> <p>(1) $-\frac{4}{3}$</p> <p>(2) $(\alpha + \beta)$ <input type="text" value="αβ"/> $(\beta + \alpha)$ <input type="text" value="αβ"/> $(-\beta^2 + 2\alpha\beta - \alpha^2)$</p> <p>2.</p> <p><input type="text" value="3"/> <input type="text" value="1"/></p>
<p style="text-align: center;">123</p>	<p style="text-align: center;">124</p>
<p>3.</p> <p>(1) <input type="text" value="2 ∫₀¹ dx"/> 4</p> <p>(2) 30</p> <p>(3) -2</p> <p>(4) $\frac{2}{5}Aa^5 + \frac{2}{3}Ca^3 + 2Ea$</p>	<p>3.</p> <p>(1) $-\frac{9}{2}$</p> <p>(2) $-\frac{4}{3}$</p> <p>(3) <input type="text" value="2"/> <input type="text" value="1/2"/> $-\frac{343}{24}$</p> <p>(4) $-\frac{343}{24}$</p>

L 125–128

125	126
<p>(1) $-\frac{5}{6}\sqrt{5}$</p>	<p>1.</p> <p>(1) 10</p> <p>(2) -10</p> <p>(3) $\frac{7}{3}$</p> <p>(4) 3</p> <p>(5) $\frac{16}{3}$</p> <p>2.</p> <p>(i) $-\int_{\boxed{0}}^{\boxed{2}}(2x+3)dx$</p> <p>(ii) $\int_{-1}^{\boxed{0}}(x^2-2x+1)dx + \int_{\boxed{0}}^3(x^2-2x+1)dx$</p>
125	126
<p>(2) $-8\sqrt{6}$</p> <p>(3) $-4\sqrt{3}$</p>	<p>3.</p> <p>(1) $\int_{\boxed{-1}}^{\boxed{2}}(2x^2-x)dx$</p> <p>$\frac{9}{2}$</p> <p>(2) $\frac{8}{3}$</p> <p>(3) 6</p>

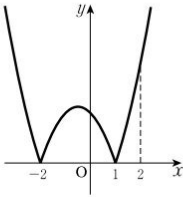
127

1.

(1) $(x+2)(x-1)$

$-x^2-x+2$	0	1
x^2+x-2	1	2

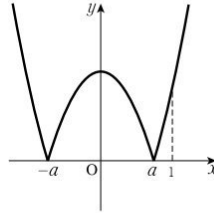
3



128

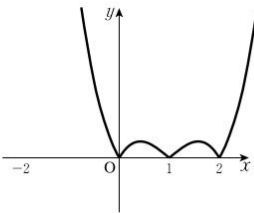
1.

$$\frac{4}{3}a^3 - a^2 + \frac{1}{3}$$



127

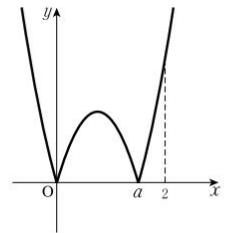
(2) $\frac{65}{4}$



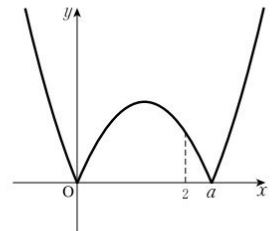
128

2.

(i) $\frac{a^3}{3} - 2a + \frac{8}{3}$



(ii) $-\frac{8}{3} + 2a$



L 129–132

129

Ex.

130

1.

(1) $\frac{9}{2}$

(2) -20

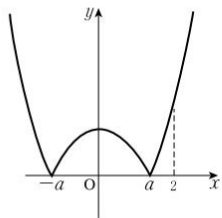
(3) $\frac{44}{3}$

129

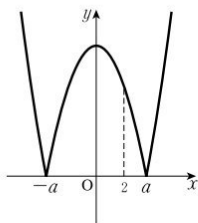
1.

The overall minimum value is 2,
when $a = 1$.

(i) When $0 < a < 2$



(ii) When $a \geq 2$



130

2.

$$-\frac{8}{3}\sqrt{2}$$

3.

$$\frac{31}{6}$$

131	132
<p>1.</p> <p>(1) $x^3 - x$</p> <p>(2) $3x^2 - 4x + 1$</p> <p>(3) $-2x^2 + 2ax$</p>	<p>(1) The relative maximum value is 27, at $x = -3$. The relative minimum value is -5, at $x = 1$.</p>
131	132
<p>2.</p> <p>(1) $-2x + 1$</p> <p>(2) $-x^2 - 2x - 3$</p> <p>(3) $-3x^2 + 4x - 1$</p>	<p>(2) The relative maximum value is $\frac{1}{4}$, at $x = 1$. The relative minimum value is 0, at $x = 0, 2$.</p>

L 133–136

133

1.

The maximum value is $\frac{4}{3}$,
at $x = 1, 4$.

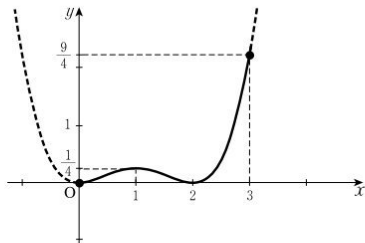
The minimum value is 0,
at $x = 0, 3$.

134

1.

The maximum value is $\frac{9}{4}$,
at $x = 3$.

The minimum value is 0,
at $x = 0, 2$.



133

2.

The maximum value is $\frac{5}{6}$,
at $x = \frac{1}{2}$.

The minimum value is -45 ,
at $x = 3$.

134

2.

$F(x)$ has a maximum value of 26,
at $x = 0$.

135	136
Ex.	Ex.
135	136
(1) $f(x) = 2x - 2$ $a = 3, -1$	(1) $f(x) = x^2 + \frac{2}{3}x$

L 137–140

137	138
<p>1.</p> $f'(x) = 2ax + b$ $f(-1) = 2$ $f'(0) = 0$ $\int_0^1 (ax^2 + bx + c) dx = -2$ $f(x) = 6x^2 - 4$	<p>Ex.</p>
137	138
<p>2.</p> $f(x) = 4x^2 - 6x + 3$	<p>1.</p> $a = 6, b = -6$

139

Ex.

140

1.

The relative maximum value is $\frac{10}{3}$,
at $x = -1$.

The relative minimum value is -18 ,
at $x = 3$.

139

1.

(1) Let $f(x) = ax + b$ (where $a \neq 0$)

$$\int_0^1 f(x) dx = \frac{a}{2} + b$$

$$\left[\int_0^1 f(x) dx \right]^2 = \frac{a^2}{4} + ab + b^2$$

$$\int_0^1 [f(x)]^2 dx = \frac{a^2}{3} + ab + b^2$$

$$\begin{aligned} \int_0^1 [f(x)]^2 dx - \left[\int_0^1 f(x) dx \right]^2 \\ = \frac{1}{12} a^2 > 0 \end{aligned}$$

Therefore,

$$\left[\int_0^1 f(x) dx \right]^2 < \int_0^1 [f(x)]^2 dx$$

140

2.

$$f(x) = 4x - 3$$

$$a = \frac{1}{2}, 1$$

L 141–144

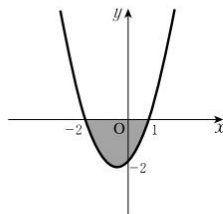
141

1.
 (1) $S = 2 \times \frac{1}{2}(4+6) = 10$

(2) $\int_2^4 (x+2)dx = 10$

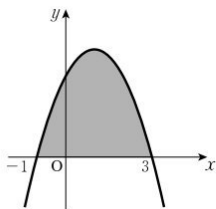
142

1.
 (1) $S = \frac{9}{2}$



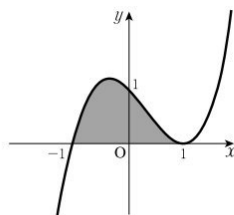
141

2.
 (1) $S = \frac{32}{3}$

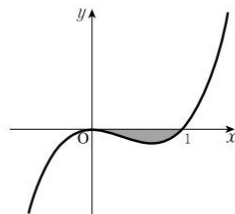


142

2.
 (1) $S = \frac{4}{3}$

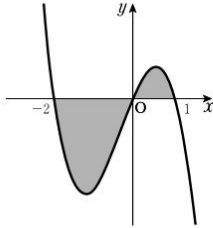


(2) $S = \frac{1}{12}$



143

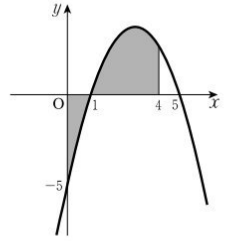
1.
 (1) $S = \frac{37}{12}$



144

1.
 $S = 4$

2.
 $S = \frac{34}{3}$



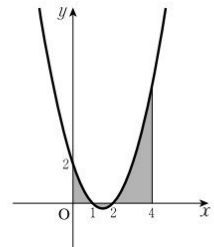
143

2.
 (1) $S = \frac{8}{3}$

(2) $S = \frac{27}{4}$

144

3.
 $S = \frac{17}{3}$



L 145–148

145

1.
(1) $\frac{22}{3}$

(2) $\frac{14}{3}$

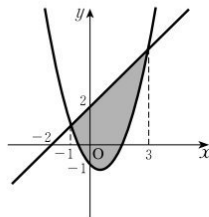
(3) $\frac{8}{3}$

2.

$g(x)$

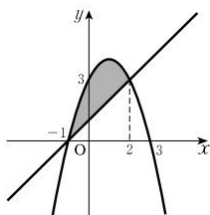
146

1.
(1) $S = \frac{32}{3}$



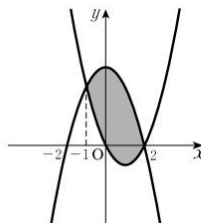
145

3.
(1) $S = \frac{9}{2}$

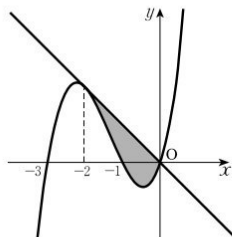


146

(2) $S = 9$



(3) $S = \frac{4}{3}$



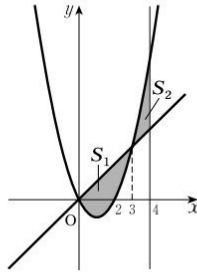
147

(1) (i) $S_1 = \frac{9}{2}$

(ii) $S_2 = \frac{11}{6}$

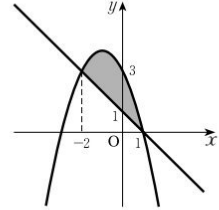
From (i) and (ii),

$$S = \frac{19}{3}$$



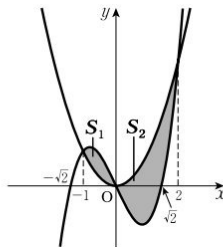
148

(1) $S = \frac{9}{2}$



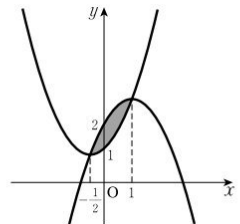
147

(2) $S = \frac{37}{12}$

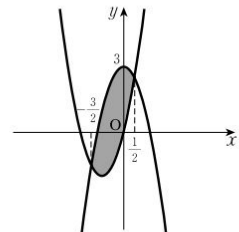


148

(2) $S = \frac{9}{8}$



(3) $S = \frac{16}{3}$



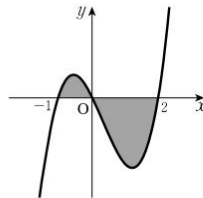
L 149–152

149

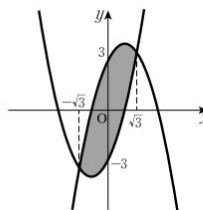
Ex.

150

(1) $S = \frac{37}{12}$

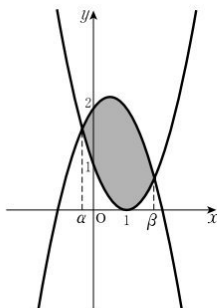


(2) $S = 8\sqrt{3}$



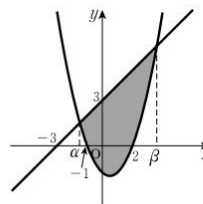
149

(1) $S = \frac{17\sqrt{17}}{24}$



150

(3) $S = 8\sqrt{6}$



151

Ex.

152

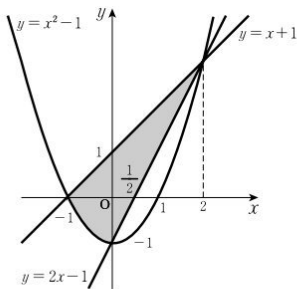
1.
 (i) $a = -3$
 (ii) $a = 3$

From (i) and (ii),
 $a = \pm 3$

151

1.

$$S = \frac{19}{6}$$



152

2.
 $a = \pm 2$

L 153–156

153

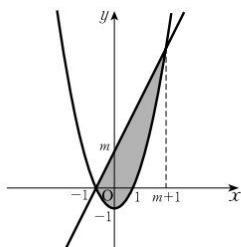
1.
 (1) $x = 0, m + 2$
 (2) $S = \frac{1}{6}(m+2)^3$
 (3) $m = 2$

154

1.
 (1) $y = 3x + 6$
 (2) $x = 2$
 (3) $S = \frac{64}{3}$

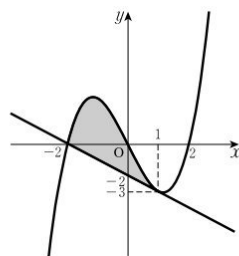
153

2.
 $m = 1$



154

2.
 $S = \frac{27}{4}$



155

1.

(1) $y = 3x + 2$

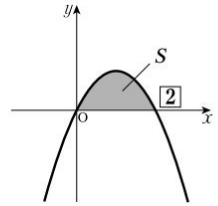
(2) $x = -1, 2$

(3) $S = \frac{27}{4}$

156

1.

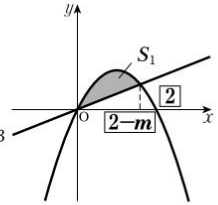
(1) $S = \frac{4}{3}$



(2) $\boxed{2}$

$\boxed{0}$ $\boxed{2}$

$S_1 = \frac{1}{6}(2-m)^3$

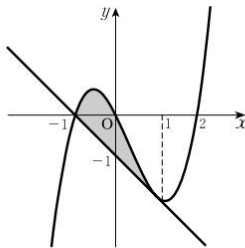


(3) $\boxed{2}$ $\boxed{4}$

155

2.

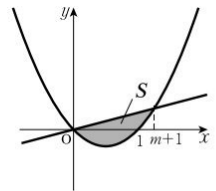
$S = \frac{4}{3}$



156

2.

$m = -1 + \sqrt[3]{2}$



L 157–160

157

1.

$$a = 1$$

158

Ex.

157

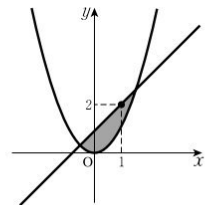
2.

$$a = 2$$

158

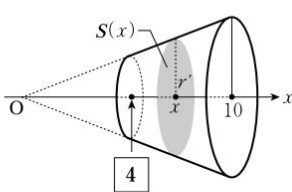
1.

$$m = 2$$



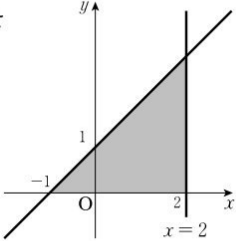
159	160																		
<p>1.</p> $S(a) = -\frac{a^4}{6} + \frac{2}{3}a^3 - \frac{4}{3}a + \frac{4}{3}$ $S'(a) = -\frac{2}{3}(a-1)(a^2-2a-2)$ <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;">a</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">...</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">...</td> <td style="padding: 2px 5px;">2</td> </tr> <tr> <td style="padding: 2px 5px;">$S'(a)$</td> <td style="padding: 2px 5px;">/</td> <td style="padding: 2px 5px;">-</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">+</td> <td style="padding: 2px 5px;">/</td> </tr> <tr> <td style="padding: 2px 5px;">$S(a)$</td> <td style="padding: 2px 5px;">/</td> <td style="padding: 2px 5px;">↘</td> <td style="padding: 2px 5px;">relative minimum</td> <td style="padding: 2px 5px;">↗</td> <td style="padding: 2px 5px;">/</td> </tr> </table> <div style="border: 1px solid black; width: 20px; height: 20px; display: flex; align-items: center; justify-content: center; margin: 10px auto;">1</div>	a	0	...	1	...	2	$S'(a)$	/	-	0	+	/	$S(a)$	/	↘	relative minimum	↗	/	<p>1.</p> $y = x+1, \quad y = -x+1$
a	0	...	1	...	2														
$S'(a)$	/	-	0	+	/														
$S(a)$	/	↘	relative minimum	↗	/														
159	160																		
<p>2.</p> $k = 3 - 2\sqrt{2}$	<p>2.</p> $S = \frac{27}{4}$																		

L 161–164

161	162
<p>Ex.</p>	<p>1.</p> <div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> x h </div> $a' = \frac{a}{h}x$ $S(x) = \frac{a^2}{h^2}x^2$ $V = \frac{1}{3}a^2h$
161	162
<p>1.</p> <div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> 5 10 </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px; display: inline-block;"> $\frac{1}{2}x$ </div> $S(x) = \frac{\pi}{4}x^2$ <div style="border: 1px solid black; padding: 2px 10px; margin-bottom: 10px; display: inline-block;">4</div> $V = 78\pi$ 	<p>2.</p> $V = 104$

163	164
1. $\frac{4}{3}\pi a^3$	1. \boxed{a} $V = \frac{2}{3}a^3$
163	164
2. $V = \frac{100}{3}\pi$	2. $S(x) = \frac{\sqrt{3}}{2}(9-x^2)$ $V = 18\sqrt{3}$

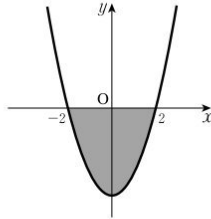
L 165–168

165	166
<p>Ex.</p>	<p>1.</p> $a^2 - x^2$ $V = \frac{4}{3}\pi a^3$
165	166
<p>1.</p> $6 \quad 9$ $V = 19\pi$ <p>2.</p> $V = 9\pi$ 	<p>2.</p> $V = 243\pi$

167

1.

$$V = \frac{512}{15}\pi$$



168

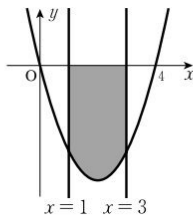
1.

$$\frac{32}{5}\pi$$

167

2.

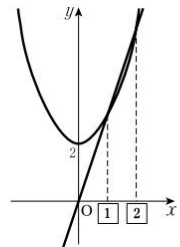
$$V = \frac{406}{15}\pi$$



168

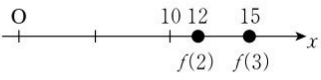
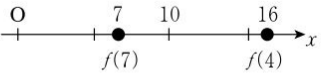
2.

$$V = \frac{22}{15}\pi$$



L 169–172

169	170
<p>1.</p> $V = \frac{107}{35}\pi$	<p>1.</p> <p>(1) $h = 2\sqrt{2}a$</p> <p>(2) $V = \frac{2\sqrt{2}}{3}\pi a^3$</p>
169	170
<p>2.</p> $V = \frac{29}{42}\pi$	<p>2.</p> $V = \frac{108}{5}\pi$

171	172
<p>1.</p> <p>(1) $x = f(1) = 7$ $x = f(1+2) = f(3) = 15$ $\frac{f(3)-f(1)}{2} = 4$ 4</p> <p>(2) </p> <p>3</p> <p>(3) </p> <p>-3</p>	<p>1.</p> <p>(1) $v = 28 - 1.12t$ $28 - 1.12t$</p> <p>(2) 28</p> <p>(3) 25</p> <p>(4) 350</p>
171	172
<p>2.</p> <p>(1) $v = 3t^2 - 6t$ $3t^2 - 6t$</p> <p>(2) $v = -3$ -3</p> <p>(3) $v = 24$ 24</p>	<p>2.</p> <p>(1) 18 m/sec</p> <p>(2) 405 m</p> <p>3.</p> <p>Maximum Velocity: 25 m/sec Distance: 625 m</p>

L 173–176

173	174
<p>1.</p> <p>(1) $v = 24.5 - 9.8t$ $24.5 - 9.8t$</p> <p>(2) 4.9</p> <p>(3) -14.7</p> <p>(4) 2.5</p> <p>(5) Time: 5 sec, Velocity: -24.5 m/sec</p>	<p>1.</p> <p>(1) $S = \frac{\pi}{25}t^2 + \frac{12\pi}{5}t + 36\pi$ $\frac{\pi}{25}t^2 + \frac{12\pi}{5}t + 36\pi$</p> <p>(2) $\frac{2\pi}{25}$ $\frac{12\pi}{5}$ $\frac{16\pi}{5}$</p> <p>(3) At time 20 seconds, 4π cm²/sec</p>
173	174
<p>2.</p> <p>(1) Velocity: 9.8 m/sec Height: 39.2 m</p> <p>(2) 44.1 m</p> <p>(3) -29.4 m/sec</p>	<p>2.</p> <p>$2 + 2t$</p> <p>$V = \frac{4}{3}\pi(2 + 2t)^3$ $= \frac{32}{3}\pi t^3 + 32\pi t^2 + 32\pi t + \frac{32}{3}\pi$ $v = 32\pi t^2 + 64\pi t + 32\pi$ 800π</p> <p>3.</p> <p>8π</p>

175	176				
<p>1.</p> $x = -8$	<p>1.</p> <p>68 cm</p>				
175	176				
<p>2.</p> $x = 5$ <p>3.</p> <p>(1) 40 m</p> <p>(2) 45 m</p>	<p>2.</p> <table border="1" data-bbox="647 906 797 1011"><tr><td>0</td><td>2</td></tr><tr><td>2</td><td></td></tr></table> <p>$s = 2$</p> <p>2 cm</p> <p>3.</p> <p>85 m</p>	0	2	2	
0	2				
2					

L 177–180

177	178
<p>1.</p> <p>(1) $x = 6t^2 - t^3$ $6t^2 - t^3$</p> <p>(2) 6 seconds later</p> <p>(3) $s = 64$</p>	<p>1.</p> <p>(1) $x_1 = 2t^2 - 12t$ $2t^2 - 12t$</p> <p>(2) $x_2 = -t^2 + 12t$</p> <p>(3) 8 seconds later</p>
177	178
<p>2.</p> <p>(1) $4t - \frac{1}{2}t^2$</p> <p>(2) $\frac{1}{2}t^2 - 4t + 16$</p> <p>(3) 10 seconds</p>	<p>2.</p> <p>(1) 3 seconds later</p> <p>(2) 8 cm</p>

179	180
<p>1.</p> <p>(1) $v = -\frac{1}{3}t^3 + 3t^2$ $-\frac{1}{3}t^3 + 3t^2$</p> <p>(2) $\frac{28}{3}$</p> <p>(3) 6</p> <p>(4) 36</p>	<p>1.</p> <p>(1) 44.1 m</p> <p>(2) -29.4 m/sec</p>
179	180
<p>2.</p> <p>(1) 63 l</p> <p>(2) 243 l</p>	<p>2.</p> <p>(1) 39 cm</p> <p>(2) 36 cm/sec</p>

L 181–184

181	182
<p>1.</p> <p>(1) $(0, 1)$ 3</p> <p>(2) $a = -1, b = 3, c = 1$</p>	<p>1.</p> <p>$a = 3, b = -12, c = 7$</p>
181	182
<p>2.</p> <p>(1) $(0, 1)$ or $\left(\frac{1}{2}, \frac{1}{8}\right)$</p> <p>(2) $a = -1, -\frac{9}{8}$</p>	<p>2.</p> <p>$a = -\frac{3}{2}, b = 0, c = \frac{1}{4}$</p>

183	184
<p>1.</p> $\begin{cases} a = 3 \\ b = -1 \end{cases} \quad \begin{cases} a = -3 \\ b = 3 \end{cases}$	<p>1.</p> <p>Maximum value: $f(-2) = 4(3a+2)$</p> <p>Minimum value:</p> $\begin{cases} f(2) = 4(3a-2) & (\text{when } 0 < a < \frac{2}{3}) \\ f(0) = 0 & (\text{when } \frac{2}{3} \leq a < 1) \end{cases}$
183	184
<p>2.</p> <p>(1) $f(2a) = -4a^3 + 2a^2 + 8a + 5$</p> <p>(2) $a = 1$</p>	<p>2.</p> <p>(i) When $0 < a < \frac{1}{2}$,</p> <p style="padding-left: 40px;">$1 - 3a^2$, at $x = 1$.</p> <p>(ii) When $\frac{1}{2} \leq a \leq 1$,</p> <p style="padding-left: 40px;">$2a^3$, at $x = a$.</p> <p>(iii) When $a > 1$,</p> <p style="padding-left: 40px;">$3a^2 - 1$, at $x = 1$.</p>

L 185–188

185	186
<p>1.</p> $a > 9$	<p>1.</p> $(1) b = \frac{1}{3}$ $(2) x = \frac{23}{9}$
185	186
<p>2.</p> $a < -1, a > 1$	<p>2.</p> $f(x) = -4x^3 + 19x^2 - 26x + 13$

187

1.

$$a = \frac{3}{2}, b = \frac{1}{2}$$

188

1.

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{5}{3}x + 1$$

$$g(x) = x^2 - x + \frac{5}{3}$$

187

2.

$$c = 2$$

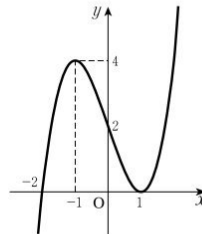
188

2.

$$(1) f'(x) = 3x^2 - 3$$

$$(2) f(x) = x^3 - 3x + 2$$

(3)



L 189–192

189	190
<p>1.</p> $S = \frac{49}{3}$	<p>1.</p> <p>(1) $y = x$</p> <p>(2) $\frac{1}{12}$</p>
189	190
<p>2.</p> $a = 3, b = 0$	<p>2.</p> <p>(1) $S_1 = 2a^4$</p> $S_2 = \frac{4}{3}a^4$ <p>(2) 3 : 2</p>

191	192
<p>1.</p> <p>(1) $y = (3a^2 - 6a + 2)x - 2a^3 + 3a^2$</p> <p>(2) $0 < p < 1$</p>	<p>1.</p> $-\frac{1}{2} \leq h \leq 0$
191	192
<p>2.</p> $\frac{1}{4} \leq k \leq \frac{3 + \sqrt{13}}{2}$	<p>2.</p> $f(x) = 5x^2$

L 193–196

193	194
<p>1.</p> <p>(1) $f(x) = x$</p> <p>(2) $g(x) = x^2 - \frac{1}{3}$</p>	<p>1.</p> <p>(1) $a = 10, b = -3$</p> <p>(2) $S = \frac{4}{3}$</p>
193	194
<p>2.</p> <p>$f(x) = \frac{10}{3}x^3 - 4x^2 + x$</p>	<p>2.</p> <p>(1) $a = 1$</p> <p>(2) $S = \frac{4}{3}$</p>

195	196
<p>1.</p> <p>(1) $y = 8x + 3$</p> <p>(2) $S = \frac{256}{5}$</p>	<p>1.</p> <p>(1) $0 < a \leq 2$</p> <p>(2) $S = \frac{a}{2} \left(\sqrt{\frac{4-a^2}{3}} \right)^3$</p>
195	196
<p>2.</p> <p>(1) $y = -4x - 4$ $y = 4x - 12$</p> <p>(2) $(1, -8)$</p> <p>(3) $S = \frac{16}{3}$</p>	<p>2.</p> <p>$S = \frac{13}{3}$</p>

L 197–200

197	198
<p>1.</p> <p>(1) $y = x - \frac{1}{4}$</p>	<p>1.</p> <p>(1) $a = 12, b = -12$</p> <p>(2) $y = -x + 6$</p>
197	198
<p>(2) $a = \frac{3}{2}$</p> <p>(3) 1 : 1</p>	<p>(3) 4 : 1</p>

199	200
<p>1.</p> $(1) \triangle ABP = -\frac{3}{2}(p^2 - p - 2)$	<p>1.</p> $S = \frac{4}{3}$
199	200
<p>(2) $p = \frac{1}{2}$</p>	<p>2.</p> $\frac{1}{2}$